Section 7.2 Solutions Math 31

2. $\int \sin^3\theta \cos^4\theta \, d\theta = \int \sin^2\theta \cos^4\theta (\sin\theta \, d\theta)$, Let $u = \cos\theta$, $du = -\sin\theta \, d\theta$, $-du = \sin\theta \, d\theta$

$= \int (1-\cos^2\theta)\cos^4\theta (\sin\theta \, d\theta)$

$= -\int (1-u^2)u^4 \, du = \int (u^6 - u^4) \, du = \frac{1}{7}u^7 - \frac{1}{5}u^5 + C$

$\therefore \int \sin^3\theta \cos^4\theta \, d\theta = \boxed{\frac{1}{7}\cos^7\theta - \frac{1}{5}\cos^5\theta + C}$

4. $\int_0^{\pi/2} \sin^5 x \, dx = \int_0^{\pi/2} \sin^4 x (\sin x \, dx)$, Let $u = \cos x$, $du = -\sin x \, dx$, $-du = \sin x \, dx$

| $x$ | $u$ |
|---|---|
| $0$ | $1$ |
| $\pi/2$ | $0$ |

$= + \int_0^{\pi/2} (1-\cos^2 x)^2 (\sin x \, dx)$

$= - \int_1^0 (1-u^2)^2 \, du = + \int_0^1 (u^4 - 2u^2 + 1) \, du$

$= \left[\frac{1}{5}u^5 - \frac{2}{3}u^3 + u \right|_0^1 = \left(\frac{1}{5} - \frac{2}{3} + 1\right) - (0 - 0 + 0)$

$= \boxed{\frac{8}{15}}$

10. $\sin^2 t \cos^4 t = \frac{1}{2}(1-\cos 2t)\left[\frac{1}{2}(1+\cos 2t)\right]^2 =$

$\frac{1}{8}(1-\cos 2t)(1 + 2\cos 2t + \cos^2 2t) =$

$\frac{1}{8}(1 + \cos 2t - \cos^2 2t - \cos^3 2t) =$

$\frac{1}{8}\left(1 + \cos 2t - \frac{1}{2}[1 + \cos 4t] - \cos^3 2t\right) =$

$\frac{1}{8}\left(\frac{1}{2} + \cos 2t - \frac{1}{2}\cos 4t - \cos^3 2t\right)$

$\therefore \int_0^{\pi} \sin^2 t \cos^4 t \, dt = \frac{1}{8}\int_0^{\pi}\left(\frac{1}{2} + \cos 2t - \frac{1}{2}\cos 4t - \cos^3 2t\right) dt$

$= \frac{1}{8}\int_0^{\pi}\left(\frac{1}{2} + \cos 2t - \frac{1}{2}\cos 4t\right) dt - \frac{1}{8}\int_0^{\pi}(1 - \sin^2 2t)\cos 2t \, dt$

$= \frac{1}{8}\left[\frac{1}{2}t + \frac{1}{2}\sin 2t - \frac{1}{8}\sin 4t \right|_0^{\pi} - \frac{1}{8}\int_0^0 (1-u^2)\left(\frac{1}{2}du\right)$

$= \frac{1}{16}t \Big|_0^{\pi}$ plus a whole lot of zeros where $u = \sin 2t$, $du = 2\cos 2t \, dt$, $\frac{1}{2}du = \cos 2t \, dt$

| $t$ | $u$ |
|---|---|
| $0$ | $0$ |
| $\pi$ | $0$ |

$= \pi/16$

Section 7.2 Solutions Continued

20. $\int \cos^2 x \sin 2x\,dx = \int \cos^2 x(2\sin x\cos x)\,dx =$

$2\int \cos^2 x \sin x(\cos x\,dx) = 2\int(1-\sin^2 x)\sin x(\cos x\,dx)$

$= 2\int(1-u^2)u\,du$, where $u = \sin x$, $du = \cos x\,dx$

$= 2\int(u-u^3)\,du$

$= 2\left(\frac{1}{2}u^2 - \frac{1}{4}u^4\right) + C = \boxed{\sin^2 x - \frac{1}{2}\sin^4 x + C}$

22. $\int \tan^2 \sec^4\theta\,d\theta = \int \tan^2\theta\sec^2\theta(\sec^2\theta\,d\theta) =$

$\int \tan^2\theta(\tan^2\theta+1)(\sec^2\theta\,d\theta) =$

$\int u^2(u^2+1)\,du$, where $u = \tan\theta$ & $du = \sec^2\theta\,d\theta$

$= \int(u^4+u^2)\,du = \frac{1}{5}u^5 + \frac{1}{3}u^3 + C = \boxed{\frac{1}{5}\tan^5\theta + \frac{1}{3}\tan^3\theta + C}$

26. $\int_0^{\pi/4} \sec^4\theta\tan^4\theta\,d\theta = \int_0^{\pi/4} \sec^2\theta\tan^4\theta(\sec^2\theta\,d\theta) =$

$\int_0^{\pi/4}(\tan^2\theta+1)\tan^4\theta(\sec^2\theta\,d\theta)$, Let $u = \tan\theta$, $du = \sec^2\theta\,d\theta$

| $\theta$ | $u$ |
|---|---|
| $0$ | $0$ |
| $\pi/4$ | $1$ |

$= \int_0^1 (u^2+1)u^4\,du = \int_0^1 (u^6+u^4)\,du$

$= \frac{1}{7}u^7 + \frac{1}{5}u^5\Big|_0^1 = \left(\frac{1}{7}+\frac{1}{5}\right) - (0+0) = \boxed{\frac{12}{35}}$

28. $\int \tan^5 x\sec^3 x\,dx = \int \tan^4 x\sec^2 x(\sec x\tan x\,dx)$

$= \int(\sec^2 - 1)^2\sec^2 x(\sec x\tan x\,dx)$, Let $u = \sec x$, $du = \sec x\tan x\,dx$

$= \int(u^2-1)^2u^2\,du = \int(u^4-2u^2+1)u^2\,du$

$= \int(u^6-2u^4+u^2)\,du = \frac{1}{7}u^7 - \frac{2}{5}u^5 + \frac{1}{3}u^3 + C$

$= \frac{1}{7}\sec^7 x - \frac{2}{5}\sec^5 x + \frac{1}{3}\sec^3 x + C$